Lesson 10: Modified 1-group Theory

- Consideration of Slowing Down
- "Roles" of p , τ_{th}
- Migration Area (M^2), Physical Significance
- L^2 , τ_{th} for a Thermal Reactor
- Schematisation of Chain Reaction
- Neutron Spectra (thermal, fast reactors)

Thermal Reactor, Considering Slowing Down

- With simple 1-group theory, n's assumed born with thermal energies ($E = E_{th}$)
- In reality, thermal neutron "sources" need to be defined:
  - $q(\vec{r}, E_{th})$ - distribution of slowing-down sources down to E_{th}
- For an infinite medium, $q(\vec{r}, E_{th}) = Q_f \cdot p$

  fission source (Q_f), reson. escape probab. = $\exp\left[-\int_{E_{th}}^{E_s} \frac{\Sigma_a(E')}{\xi\,\Sigma_s(E')}\frac{dE'}{E'}\right]$ (p)
- For the finite system, one may consider the leakage during slowing down as absorption with $\Sigma_a'(E') = D(E')B^2$
- Thus, $q(\vec{r}, E_{th}) = Q_f \cdot \exp\left[-\int_{E_{th}}^{E_s} \frac{[\Sigma_a(E') + D(E')B^2]}{\xi\,\Sigma_s(E')}\cdot\frac{dE'}{E'}\right]$

  $= Q_f \cdot p \cdot e^{-B^2\tau_{th}}$ with $\tau_{th} = \int_{E_{th}}^{E_s} \frac{D(E')}{\xi\,\Sigma_s(E')}\cdot\frac{dE'}{E'}$ (Fermi age-to-thermal, or slowing-down area)

Thermal Reactor, Considering Slowing Down (contd.)

- Assuming that only thermal n's induce fission $\Rightarrow \quad Q_f = (\bar{\nu}\Sigma_f)_{th} \cdot \phi_{th}(\vec{r})$ — thermal flux
  - Thus,

$$q(\vec{r}, E_{th}) = (\bar{\nu}\Sigma_f)_{th} \cdot p \cdot e^{-B^2\tau_{th}} \cdot \phi_{th}(\vec{r})$$

reson. escape prob. (p) — non-leakage prob. for slowing-down n's, $P_{NF,\,f}$ ($e^{-B^2\tau_{th}}$)

- Using this expression for the thermal neutron source in the 1-group equation,

$$D_{th} \cdot \nabla^2\phi_{th} - (\Sigma_a)_{th}\phi_{th} + (\bar{\nu}\Sigma_f)_{th} \cdot p \cdot e^{-B^2\tau_{th}} \cdot \phi_{th} = 0$$

i.e.

$$\nabla^2\phi_{th} + \left[\frac{k_\infty e^{-B^2\tau_{th}} - 1}{L^2_{th}}\right] \cdot \phi_{th} = 0 \qquad \text{---(1)}$$

with

$$k_\infty = \left(\frac{\bar{\nu}\Sigma_f}{\Sigma_a}\right)_{th} \cdot p = (\eta \epsilon f)_{th} \cdot p$$

Thermal Reactor, Considering Slowing Down (contd.2)

- If one considers that there is a slight increase in k_∞ due to fast fissions,

$$k_\infty = (\eta_c f)_{th} \cdot p \cdot \varepsilon \qquad \text{---}(2)$$

where $\varepsilon > 1$ (fast fission factor)

- (2) is the Four-factor Formula
- (1) , i.e. $\nabla^2 \phi_{th} + \left[\frac{k_\infty e^{-B^2 \tau_{th}} - 1}{L_{th}^2} \right] \cdot \phi_{th} = 0$

$\Rightarrow$ *Modified 1-group Reactor Equation*

with Criticality Condition:

$$\left[\frac{k_\infty e^{-B^2 \tau_{th}} - 1}{L_{th}^2} \right] = B^2$$

B_m^2 — (the bracketed term)

B^2 — smallest eigenvalue (geometrical buckling)

Comments

- For the critical state, one has: $k_\infty = (1 + L_{th}^2 B^2) \cdot e^{B^2 \tau_{th}}$

  i.e. $k_{eff} = 1 = k_\infty \cdot \left(\frac{1}{1 + L_{th}^2 B^2} \right) \cdot e^{-B^2 \tau_{th}}$

  where $\frac{1}{1 + L_{th}^2 B^2} = P_{NF,th}$ and $e^{-B^2 \tau_{th}} = P_{NF,f}$

- With p = 1 , τ_{th} = 0 (no loss of n's during slowing down), one has the preceding (simple 1-group) equation

- Distribution $\phi(\vec{r})$, and hence the slowing-down sources, depend (as before) on the first eigenfunction of: $\nabla^2\phi + B^2\phi = 0$

Comments (contd.)

- For a non-critical reactor, $k_{eff} = \dfrac{k_\infty \, e^{-B^2 \tau_{th}}}{1 + L_{th}^2 B^2} \neq 1$
  - One may, as before, use the critical "formalism" for identifying the modification needed to have a critical system (… fictitious medium emitting $\dfrac{\bar{\nu}}{k_{eff}}$ n's per fission)
- In spite of the wide range of energies the n's have in the reactor (> 10 MeV to < 0.01 eV), one has been able to arrive at a 1-group representation, thanks to…
  - Events at thermal energies are the most important in a thermal reactor (80 to 90% of absorptions, typically), and it is possible to obtain well-defined cross-sections…
  - Slowing-down process described in relatively simple manner
    - p : resonance absorptions in fertile material , τ_{th} : leakage of fast n's
- η_c , f determined by thermal data, e.g. f (thermal utilisation factor) = $\left(\dfrac{\Sigma_{ac}}{\Sigma_{ac} + \Sigma_{am}} \right)_{th}$
- p , τ_{th} depend on epithermal characteristics

Migration Area, M^2

- Usually $L_{th}^2 B^2$, $B^2 \tau_{th}$ are small, so that

$$k_\infty = (1 + L_{th}^2 B^2) \cdot e^{B^2 \tau_{th}} \cong (1 + L_{th}^2 B^2) \cdot (1 + \tau_{th} B^2)$$

$$\cong 1 + (L_{th}^2 + \tau_{th}) B^2 = 1 + M^2 B^2$$

with M^2 (*migration area*) $= L_{th}^2 + \tau_{th}$

- One arrives at another form of the modified 1-group reactor equation :

$$\nabla^2 \phi_{th} + \left[\frac{k_\infty - 1}{M^2} \right] \phi_{th} = 0$$

with the critical condition: $B_m^2 = \dfrac{k_\infty - 1}{M^2} = B^2$

- Effectively, the entire 1-group formalism has been preserved
  - One has simply replaced L_{th}^2 by $M^2 = L^2 + \tau_{th}$

Comments

- One had:

$$L_{th}^2 = \frac{1}{6} \langle r^2 \rangle$$

average square of the distance travelled by a thermal neutron before being absorbed

and

$$\tau_{th} = \frac{1}{6} \langle r^2 \rangle$$

average square of the distance between emission at E ~ 2 MeV and slowing down to E_{th}

- Thus, $M^2 \propto \langle r^2 \rangle$ — average square of the total distance travelled by a fission neutron - during slowing down, as well as during diffusion as a thermal neutron

- $$P_{NF,th} = \frac{1}{1 + L_{th}^2 B^2} \quad , \quad P_{NF,r} \cong \frac{1}{1 + \tau_{th} B^2}$$

$$\Rightarrow \quad (P_{NF})_{tot} = P_{NF,th} \cdot P_{NF,r} \cong \frac{1}{1 + M^2 B^2}$$

L_{th}^2 , τ_{th} for a Homogeneous Reactor

- $$L_{th}^2 = \frac{D_{th}}{(\Sigma_a)_{th}} \cong \frac{(D_m)_{th}}{(\Sigma_{ac} + \Sigma_{am})_{th}}$$ (very low concentration of fuel)

$$= \left(\frac{D_m}{\Sigma_{am}}\right)_{th} \cdot \left(\frac{\Sigma_{am}}{\Sigma_{ac} + \Sigma_{am}}\right)_{th} = \underbrace{\left(\frac{D_m}{\Sigma_{am}}\right)_{th}}_{(L_m^2)_{th}} \cdot \left[1 - \underbrace{\left(\frac{\Sigma_{ac}}{\Sigma_{ac} + \Sigma_{am}}\right)_{th}}_{f}\right]$$

$\Rightarrow$ $$L_{th}^2 = (L_m^2)_{th} \cdot [1 - f]$$

- $\tau_{th} \sim (\tau_m)_{th}$ ... low fuel concentration

$$\cong \int_{E_{th}}^{E_s} \frac{D(E')}{\xi \Sigma_s(E')} \frac{dE'}{E'}$$ $\Rightarrow$

Moderator	ξ	τ_{th} (cm^2)
H_2O	0.92	27
D_2O	0.509	131
C	0.158	368

Comments

- L_{th}^2 depends on thermal cross-sections → $L_{th}^2 = (L_m^2)_{th} \cdot [1-f]$
- The slowing-down "constants" depend on the moderator characteristics

$$p = \exp\left[-\frac{N_c I_{eff}(\sigma_e)}{\xi (N_m \sigma_m + N_c \sigma_{sc}^*)}\right] \qquad \tau_{th} = \int_{E_{th}}^{E_s} \frac{D(E')}{\xi \Sigma_s(E')} \frac{dE'}{E'}$$

  - E.g., for high value of ξ, p ↑ and τ_{th} ↓ (losses ↓)
- H_2O, most "powerful" moderator ⇒ slowing-down power: $\xi \Sigma_s$
- However, Σ_a also important ⇒ moderating ratio: $\frac{\xi \Sigma_s}{\Sigma_a}$
- D_2O, graphite offer much better "neutronic compromise"
  - One can have a critical reactor* with natural uranium

(*not homogeneous, though…)

Summary: Bare, Homogeneous Reactors

- (Critical) Reactor Equation: $\nabla^2\phi + B_m^2\phi = 0$

Simple 1-group: $B_m^2 = \frac{k_\infty - 1}{L^2}$

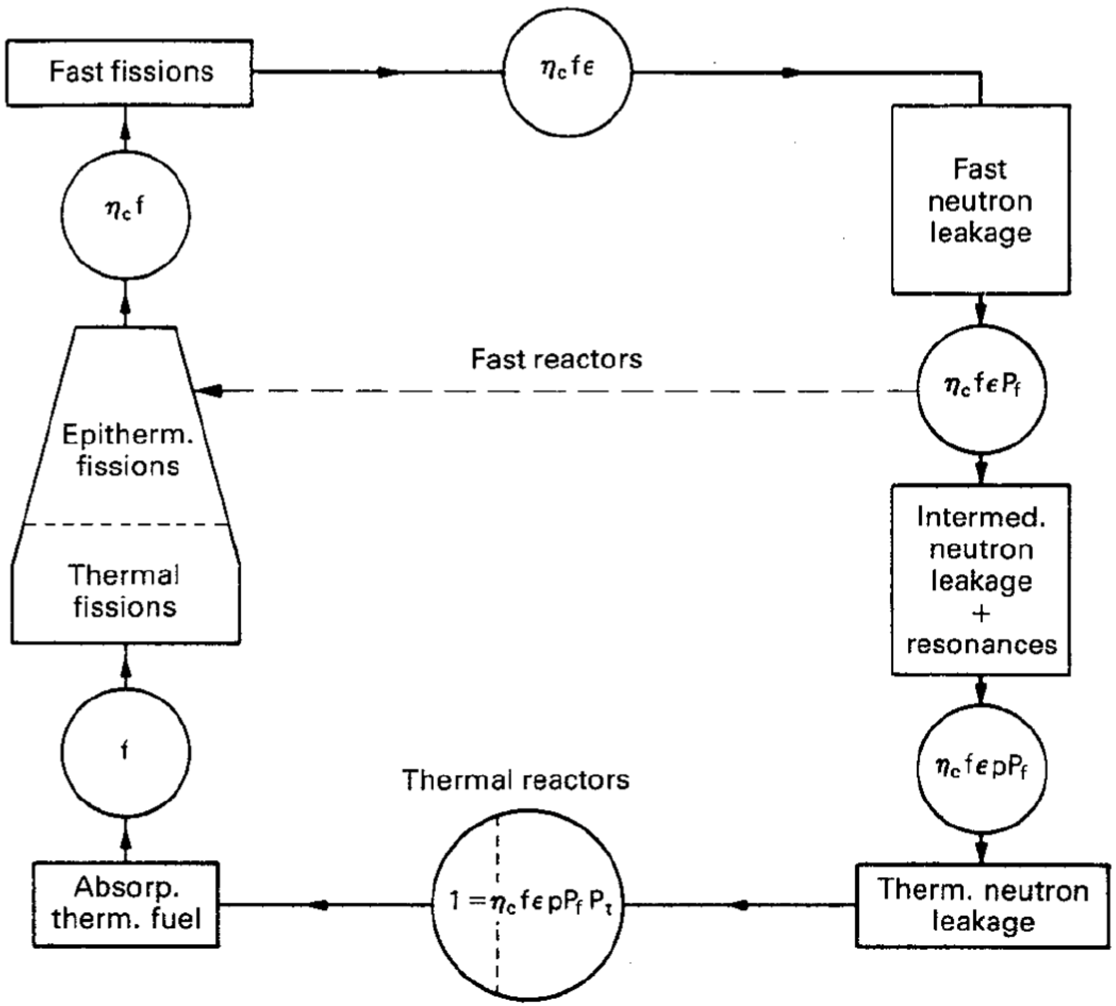
Modified 1-group (thermal reactor): $B_m^2 = \frac{k_\infty e^{-B^2\tau_{th}} - 1}{L_{th}^2} \cong \frac{k_\infty - 1}{M^2}$

- Criticality condition: $B_m^2 = B^2$

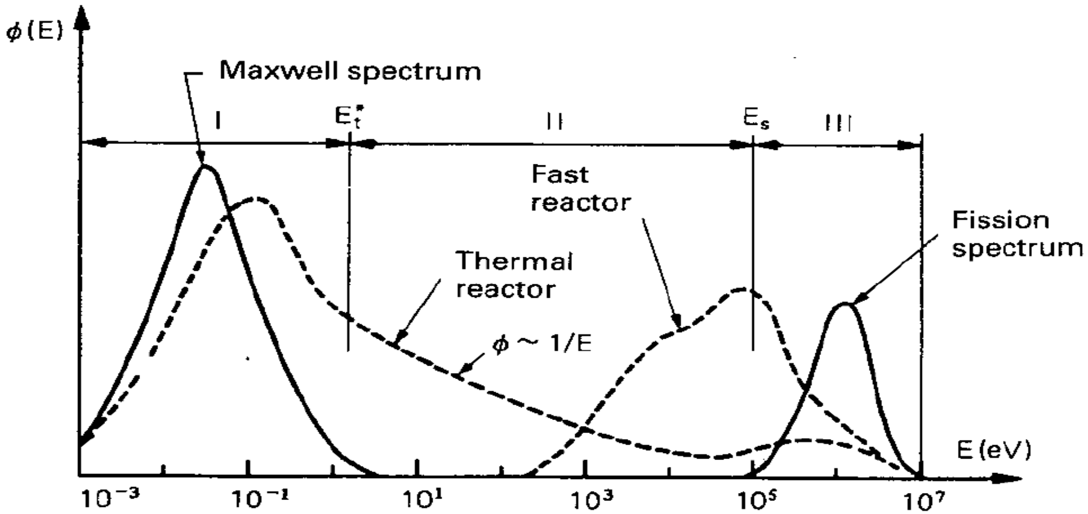
material buckling (B_m^2); geometrical buckling (smallest eigenvalue) (B^2)

- Spatial distribution of flux: eigenfunction corresponding to…
- Absolute flux: Coefficient A determined by reactor power P
- Ratio $\overline{\Phi} / \Phi_{max}$ … independent of P

Schematisation of Chain Reaction



Neutron Energy Spectra (repeat from Lesson 3)



- For a thermal reactor (easiest establishing of a chain reaction with ~95% of fissions "thermal")
  - Region I… close to a Maxwellian spectrum, somewhat distorted
    - harder (due to higher absorptions at low energies), softer (due to greater leakage of fast neutrons)
  - Region II… slowing down region ($\phi \sim 1/E$)
  - Region III… Fast region ("degraded" fisson spectrum)
- For a fast reactor, very specific goal (breeding)
  - Moderation avoided as far as possible (E_{avg} ~ between in 0.1 - 0.2 MeV range)

Summary, Lesson 10

- Simplified consideration of slowing down
- "Roles" of p , τ_{th}
- Modified 1-group Reactor Equation
- Migration area (M^2), physical significance
- L^2 , τ_{th} for a thermal reactor
- Schematisation of chain reaction
- Neutron Spectra (thermal, fast reactors)